
1- Discrete Random variables (for $0 \leq p \leq 1$, we use $q := 1 - p$)

Bernoulli: $X \sim \text{Bernoulli}(p)$

$$\mathbb{P}(X = 1) = p \quad \text{and} \quad \mathbb{P}(X = 0) = q.$$
$$\mathbb{E}(X) = p, \quad \text{Var}(X) = pq, \quad M_X(t) = q + pe^t.$$

Binomial: $X \sim \text{Binomial}(n, p)$, $n \in \mathbb{N}$.

$$\mathbb{P}(X = k) = \binom{n}{k} q^{n-k} p^k, \quad k = 0, 1, \dots, n.$$
$$\mathbb{E}(X) = np, \quad \text{Var}(X) = npq, \quad M_X(t) = (q + pe^t)^n.$$

Geometric: $X \sim \text{Geometric}(p)$

$$\mathbb{P}(X = k) = q^{k-1} p, \quad k = 1, 2, \dots$$
$$\mathbb{E}(X) = \frac{1}{p}, \quad \text{Var}(X) = \frac{q}{p^2}, \quad M_X(t) = \frac{pe^t}{1 - qe^t}.$$

Poisson: $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$.

$$\mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, \dots$$
$$\mathbb{E}(X) = \lambda, \quad \text{Var}(X) = \lambda, \quad M_X(t) = e^{\lambda(e^t - 1)}.$$

Negative Binomial: $X \sim \text{NB}(n, p)$, $n \in \mathbb{N}$.

$$\mathbb{P}(X = k) = \binom{k-1}{n-1} p^n q^{k-n}, \quad k = n, n+1, \dots$$
$$\mathbb{E}(X) = \frac{n}{p}, \quad \text{Var}(X) = \frac{nq}{p^2}, \quad M_X(t) = \left(\frac{pe^t}{1 - qe^t} \right)^n.$$

2- Continuous Random variables

Uniform: $X \sim U(a, b)$, where $a < b$.

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0 & \text{otherwise} \end{cases}$$
$$\mathbb{E}(X) = \frac{a+b}{2}, \quad \text{Var}(X) = \frac{(b-a)^2}{12}, \quad M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)}, \quad t \neq 0.$$

Exponential: $X \sim \text{Exp}(\lambda)$, where $\lambda > 0$.

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0 & \text{otherwise} \end{cases}$$
$$\mathbb{E}(X) = \frac{1}{\lambda}, \quad \text{Var}(X) = \frac{1}{\lambda^2}, \quad M_X(t) = \frac{\lambda}{\lambda - t}, \quad t < \lambda.$$

Normal: $X \sim N(\mu, \sigma^2)$, where $\mu, \sigma \in \mathbb{R}$.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad -\infty < x < \infty.$$
$$\mathbb{E}(X) = \mu, \quad \text{Var}(X) = \sigma^2, \quad M_X(t) = e^{\mu t + \frac{(\sigma t)^2}{2}}.$$

Cauchy:

$$f_X(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$
$$\mathbb{E}(X) = \text{undefined}, \quad \text{Var}(X) = \text{undefined}.$$

Gamma: $X \sim \Gamma(\omega, \lambda)$, where $\omega, \lambda > 0$.

$$f_X(x) = \begin{cases} \frac{1}{\Gamma(\omega)} \lambda^\omega x^{\omega-1} e^{-\lambda x}, & x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad \Gamma(\omega) = \int_0^\infty x^{\omega-1} e^{-x} dx.$$
$$\mathbb{E}(X) = \frac{\omega}{\lambda}, \quad \text{Var}(X) = \frac{\omega}{\lambda^2}, \quad M_X(t) = \left(\frac{\lambda}{\lambda - t} \right)^\omega, \quad t < \lambda.$$
